ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key message

In order to succeed in this paper, candidates need to have a full understanding of all the topics on the syllabus. They need to read the questions carefully in order to ensure that they are answering the question asked, using the required method and giving their answer in the form required, where these are specified.

General comments

Most candidates were able to attempt all the questions with varying levels of success and timing did not appear to be an issue. For some candidates, the use and simplification of expressions involving brackets was poor throughout the paper, leading to a loss of accuracy marks. Similarly, understanding of the $f'(x)$ notation caused problems for some candidates.

There were clearly some candidates who were very well prepared for the examination, showing a good understanding of the syllabus and its applications. Other candidates were not so well prepared especially when it came to questions that needed a little more thought rather than just the application of a routine method or formula.

Candidates should also be encouraged to take care with the level of accuracy they work with and also, unless instructed otherwise, to give their answers to 3 significant figures as specified on the front of the paper.

Candidates mostly set their work out within the confines provided by the question paper. Those candidates that find that they do not have enough room to answer a particular question should continue their answers on additional sheets and not elsewhere within the body of the examination booklet, unless there are blank pages at the end. Additional sheets should only be used for this purpose and not issued as a matter of course. Candidates should not attach blank additional sheets to their examination booklets.

Comments on specific questions

Question 1

(i) Many candidates either failed to see the modulus notation or did not understand what it meant as the most common answer for this part of the question was $\begin{pmatrix} 24 \\ 7 \end{pmatrix}$. This was given some credit, but candidates should be encouraged to read each question carefully.

(ii) The great majority of candidates correctly equated like vectors and formed two simultaneous equations. However, sign errors were fairly common leading to a loss of accuracy marks when it came to solving these simultaneous equations.

Answer: **(i)** 25 **(ii)** $\lambda = 4$, $\mu = -5$

Question 2

(i) This was usually done well, with most candidates gaining full marks.

(ii) It was pleasing to note that most candidates did take note of the word 'Hence' and made use of their matrix from part **(i)**. Candidates should be encouraged to realise the importance of the order in which matrix multiplication is done. Many candidates, having realised that they had to use the matrix that they had found in part **(i)**, post-multiplied with this matrix rather than use the correct pre-multiplication.

Answer: **(i)** $\frac{1}{2}\begin{pmatrix}1.5 & 1\\ 1 & 2\end{pmatrix}$ **(ii)** $\begin{pmatrix}0.5 & 6.5\\ 0 & 7\end{pmatrix}$

Question 3

(i) Many candidates obtained full marks on this question showing clearly each step of the solution. The majority of these candidates changed $\cot\theta$ to $\frac{\cos\theta}{\sin\theta}$. Most candidates were then successful in writing the expression as a single fraction with a correct common denominator. The use of the correct identity was handled correctly in most cases. Cancelling $(1+\cos\theta)$ frequently looked unclear and benefit of doubt was often given when candidates had used brackets. Where candidates had not used brackets, cancelling of just the $\cos\theta$ term was sometimes penalised if the resulting expression cast doubt on the candidate's intentions. This question was an example of the poor use of brackets mentioned in the general comments.

Where candidates had used an alternative method using $\tan\theta$, they often failed to achieve more than the first method mark for writing the expression as a single fraction.

(i) Most candidates realised that they needed to make use of their result from part **(i)**. Where candidates stated that $\sin\theta = 2$ or $\text{cosec}\,\theta = \frac{1}{2}$ they usually went on to give a correct reason as to why there was no solution. Where candidates did not quantify the $\sin\theta$, but merely stated a reason, these reasons were often very ambiguous and no credit could be given.

Question 4

Provided candidates had a good basic knowledge of the rules involving logarithms, both parts of this question were usually done well.

(i) Many candidates were able to rewrite the first equation in a correct logarithmic form, with the second equation usually being more problematic. Many candidates did not realise that the two resulting equations could be solved simultaneously. Solutions involving the index form of the given equations usually followed the same pattern, with two equations in index form being given correctly and some problems occurring when solutions of these equations were attempted.

(ii) Many candidates did not attempt this part if they had had problems obtaining an answer for part **(i)**. For those that did, most realised a change of base was needed and were often able to gain credit for this even if they had not been successful in part **(i)**.

Answer: **(i)** $\log_a p = 6$, $\log_a q = 3$ **(ii)** 0.5

Question 5

Most candidates were able to make a good attempt at this question, showing a sound understanding of the mathematical techniques involved. Sign errors in substitution and the subsequent solving of the resulting quadratic equation were the usual sources of error, although candidates were able to gain method marks in these cases. Some candidates having found the coordinates of *A* and of *B*, then omitted to complete the question by failing to find the length of the line *AB*.

Answer: 17.9 or equivalent.

Question 6

Candidates found this question one of the most difficult on the paper. In many cases, problems started when candidates miscopied the one side of the triangle as $4\sqrt{3}+1$, rather than the given $4\left(\sqrt{3}+1\right)$. This error was sometimes made at the beginning of the question, sometimes part way through the question. Candidates should be encouraged to take care when reading the question and also with their own subsequent workings.

Most candidates realised that the sine rule was involved and made a reasonable attempt to use it. Problems arose with the simplification of the surd expressions. Many errors were made in the multiplication out of the brackets involving surds, although the great majority of candidates were able to make a reasonable attempt to use rationalisation, for which they were usually given credit.

Some candidates mistakenly thought that the triangle was a right-angled triangle, even though no mention had been made of this. Candidates should be encouraged not to make assumptions that are not backed up by given facts.

It was pleasing to see that most candidates did attempt this quite difficult question using surds, and did not resort to the use of a calculator.

Answer: $6-4\sqrt{2}$

Question 7

(i) Candidates needed to formulate a basic strategy in attempting to answer this rather unstructured question, and a pleasing number did just that. Problems arose if candidates did not realise that the *y*-coordinate of the point *D* was 8. For those candidates that did realise this, many went on to find the equations of the appropriate lines and find the point of intersection of these lines which gave the coordinates of the point *E*. Finding the *x*-coordinate of the point *D* caused the most problems, with many candidates appearing to 'guess' it from the given diagram. Such candidates were unable to gain any accuracy marks as they had not shown sufficient working.

(ii) Candidates used both the matrix method for the area and the formula for the area of a trapezium in equal proportion. For those candidates who had not realised that the *y*-coordinate of *D* was 8, the area of the figure they were finding was not a trapezium so credit could not be given.

Answer: **(i)** *D* (3, 8), *E*(5.4, 9.2) **(ii)** 32

Question 8

Many candidates were able to make a good attempt at this question provided they had a sound knowledge of radian measure. There were many candidates who gained full marks for this question, although some lost accuracy marks due to premature approximation.

(i) Most candidates were able to use the sine rule for the area of a triangle and also the appropriate formula for the area of a sector. Problems arose only if the incorrect lengths were used. Those candidates that tried to use degrees were usually less successful. Candidates should be encouraged not to try to use degrees when dealing with questions which are clearly designed to test knowledge of radians.

(ii) As for part **(i)**, many candidates were able to make a reasonable attempt. Most realised that use of the cosine rule was the quickest way to find the length of the side *BD*, although other trigonometric methods were just as successful and acceptable. Some candidates erroneously thought that the triangle was equilateral, assigning a length of 18 to *BD*. Most were able to find the appropriate arc length and make a reasonable attempt at the perimeter required.

Answer: **(i)** 86.6, **(ii)** 55.5

Question 9

(a) (i) Many candidates were able to produce graphs with varying degrees of success. Some were unable to deal with the multiple angles correctly, but had correct amplitudes and vice versa. There were many correct graphs of $y = \sin 2x$, but problems arose with the drawing of $y = 1 + \cos 2x$. Many candidates drew it as a V shape, rather than a curve which started and finished at a maximum point.

(ii) The intention of the question was to have the candidates find the points of intersection of their graphs, which many did. It was also acceptable to give the solutions to the given equation by inspection, which many candidates, having not done too well in part **(i)** did do.

(b) (i) Usually done well, most candidates were able to gain at least one mark by recognising either the amplitude or the period of the given function.

(ii) Very few candidates obtained the correct answer to this part of the question as most assumed erroneously that the period of $y = \tan x$ is 2π **,** rather than the correct answer of π .

Answer: **(a)(ii)** $\frac{\pi}{4}$, $\frac{\pi}{2}$ **,** **(b)(i)** 5, $\frac{\pi}{2}$ **,** **(ii)** $\frac{\pi}{3}$

Question 10

(i) Done with varying degrees of success, this part of the question was dependent on whether candidates recognised the notation f′(x). Many mistook the notation to mean the inverse of the given function and thus were unable to obtain full marks for this part. Most were able to obtain credit for substitution of $x = \frac{1}{2}$ into the given function, but many, if they were unable to continue, did not equate this to zero. The value of b was given so that candidates would have a chance to continue with the rest of the question should they be unable to complete part **(i)** in its entirety.

(ii) This was usually done well with candidates making use of either algebraic long division or the remainder theorem, with equal success, to obtain at least the method mark available for this part.

(iii) Most candidates were at least able to obtain the method mark available for this part by using algebraic long division, equating coefficients or by observation, all of which were done with equal success.

(iv) Most candidates recognised that one solution of the equation was $x = \frac{1}{2}$, with many also obtaining the second solution correctly. It was not expected that candidates need mention that $x = \frac{1}{2}$ was a repeated root.

Answer: **(i)** $a = -7$, **(ii)** –49, **(iii)** $(2x - 1)(2x^2 + 3x - 2)$, **(iv)** $x = \frac{1}{2}$, –2

Question 11

This is the last session where there will be an option in the final question. The Either option was marginally more popular than the Or option, with both being completed with comparable success.

Either

(i) Most candidates gained full marks for this part of the question by making correct use of the differentiation of a quotient.

(ii) Most realised that they had to solve $\frac{dy}{dx} = 0$, which was usually done successfully although there were candidates who mistakenly thought that $10x = (1+x^2)^2$, rather than the correct $10x = 0$. Most candidates chose to find the second derivative in order to determine the nature of the turning point. To gain the final accuracy mark in this part, it was expected that candidates would be working with a completely correct expression for the second derivative as fortuitous answers were easy to obtain.

(iii) Questions are now written in this form so that candidates that have the option of numerical integration on their calculator to not have an advantage over those candidates that do not. Many candidates did not use their answer to part **(i)** to help them, even though instructed to, choosing instead to use spurious methods of integration to obtain completely incorrect results. Candidates should be encouraged to practice this type of question involving ‘reverse differentiation’.

There were also a pleasing number of candidates who did produce a completely correct solution to this part of the question.

Answer: **(i)** $\frac{10x}{(1+x^2)^2}$, **(ii)** (0,0) minimum, **(iii)** 0.15

Or

(i) Most candidates gained 3 marks for this part of the question by making correct use of the differentiation of a quotient. Problems arose with misuse of brackets and subsequent algebraic simplification, which lost many candidates the final mark. This final mark was awarded strictly as the candidates had a given answer to work towards.

(ii) Use of the given conditions at least once was made by most candidates, with many obtaining both method marks available, by using the given conditions a second time and attempting to solve the two equations obtained. Sign errors were frequently made resulting in few correct values for *A* and for *B* being found.

(iii) Most realised that they had to solve $\frac{dy}{dx} = 0$, which was usually done successfully although there were candidates who were unable to solve the equation correctly. Some candidates also forgot to find the corresponding value for *y*. Most candidates chose to find the second derivative in order to determine the nature of the turning point. To gain the final accuracy mark in this part, it was expected that candidates would be working with a completely correct expression for the second derivative as fortuitous answers were easy to obtain.

Answer: **(ii)** $A = 2$, $B = 1$, **(iii)** $\left(0, -\frac{1}{2}\right)$, maximum

ADDITIONAL MATHEMATICS

Paper 4037/13

Paper 13

Key Messages

In order to succeed in this paper, candidates need to have a full understanding of all the topics on the syllabus. They need to read the questions carefully in order to ensure that they are answering the question asked, using the required method and giving their answer in the form required, where these are specified.

General Comments

Most candidates were able to attempt a high proportion of the questions and the standard of presentation was very good.

Candidates should be aware that they should work with four or more figures in order to be able to give answers correct to three significant figures, as requested in the rubric. This paper required candidates to use angles measured in radians in several questions and candidates would benefit from practice in this area. In particular, effective use should be made of a calculator in order to work in radians rather than attempting to convert between degrees and radians. Candidates should be aware that the formulas for differentiation of products and quotients are not given in the formula list on the paper.

Comments on Specific Questions

Question 1

(a) Both parts were answered well with most candidates obtaining at least one mark. The second part was less well done with some candidates having difficulty identifying the intersection and so shading extra areas.

(b) (i) A good number of each of the various acceptable answers to this part were seen. However, candidates were less comfortable with this part of the question than the other parts. Although clearly familiar with set operators, candidates found it difficult to use them to express a logical statement, sometimes having B and F the wrong way round, or not expressing a relationship at all; for example, writing $B \cap F$ without setting it equal to F. A common misunderstanding was to use the 'is a member of' symbol instead of the 'is a subset of' symbol.

(ii) Most candidates recognised that $S \cap F$ was the relevant set and many gave one of the two correct relationships. Candidates should be aware that care and full understanding of set notation are required to avoid incorrect statements such as $S \cap F = 0$, $n(S \cap F) = \varnothing$ and $S \cap F = \{0\}$.

Answers: **(ii)** $F \subset B$ **(iii)** $S \cap F = \varnothing$

Question 2

(i) Candidates who found difficulty with this question did so for a variety of reasons. Many worked in degrees and candidates should be aware of the need to ensure that their calculator is in the correct mode when evaluating trigonometric functions. Some candidates seemed unsure how to handle a squared trigonometric function. Others seemed unsure about the order of operations. A function of t that was a rate of change was given in the question but some candidates unnecessarily attempted to differentiate the given function of t to find a rate of change.

(ii) Many candidates misunderstood what was required in this part and a significant proportion went no further than evaluating $3\sin t$, which was insufficient to earn marks. Of those who realised that the chain rule was required many multiplied $\frac{dy}{dt}$ by $\frac{dx}{dt}$ rather than dividing by $\frac{dx}{dt}$. As in the first part there were candidates who thought that differentiation of $3\sin t$ was required.

Answers: **(i)** 3 **(ii)** 0.5

Question 3

In all parts of this question, very few candidates mistakenly used permutations rather than combinations.

(i) This part was particularly well answered.

(ii) This part was also well answered. A few candidates added the two combinations rather than multiplying them.

(iii) Many candidates obtained the correct answer by adding six products of combinations without spotting the shorter method of subtracting 36 from 6435. Some candidates answered the question 'at most one woman' rather than 'at least one woman'.

Answers: **(i)** 6435 **(ii)** 1890 **(iii)** 6399

Question 4

(i) Candidates were generally less successful in sketching $y = \tan x$ than $y = 1 + 3\sin 2x$. Candidates, helped by the provision of a grid, often made a good start by plotting key points. However, sketches of $y = \tan x$ were not always continued beyond a last plotted point and so the nature of the asymptote at $x = \frac{\pi}{2}$ was not demonstrated. Candidates also had to appreciate that $y = \tan x$ did not cross the line $x = \frac{\pi}{2}$. There were many good attempts at $y = 1 + 3\sin 2x$ that appreciated the essential shape of the curve, but it was a common error to finish at $(\pi, 0)$ rather than $(\pi, 1)$. A significant number of candidates lost marks in all parts when one or both of their curves were consistent with an x-axis labelled to 2π rather than π. Candidates would benefit from practice in thinking in radians and using a calculator set in radian mode when appropriate.

(ii) Candidates with a good graph of $y = 1 + 3\sin 2x$ in part **(i)** went on to identify both points successfully. Those who tried differentiation were less successful, often losing accuracy or forgetting to find the y-coordinates.

(iii) Most candidates appreciated that the number of solutions corresponded to the number of intersections on their graph. Candidates should be aware that when asked for a number of solutions they will not be required to calculate actual solutions.

Answers: **(ii)** $\left(\frac{\pi}{4}, 4\right)$, $\left(\frac{3\pi}{4}, -2\right)$ **(iii)** 3

Question 5

(i) Successful solutions in both parts of this questions relied on commencing with a carefully drawn triangle consisting of a horizontal vector for the wind following on from a vector representing the speed in still air. Many candidates did not appreciate that it was the third side of this triangle that was on a bearing of 030°. Candidates with a good diagram often went on to score well but there were other candidates who presented sound working using the sine rule and/or the cosine rule and who could have scored well with a correct diagram derived from a better understanding of relative velocity. Candidates obtaining a correct angle within the triangle sometimes forgot to use it to calculate a bearing.

(ii) Most candidates appreciated that time is equal to distance divided by speed but no progress could be made in this part without a speed derived from a vector triangle. Candidates with full marks in part **(i)** usually went on to obtain full marks in this part by finding a side of their triangle, but care was required in the calculation of the third angle.

Answers: **(i)** 042.5° **(ii)** 1.65 hours

Question 6

(i) This question was very well answered with most candidates able to select the appropriate terms from the binomial expansion and form an equation.

(ii) Most candidates obtained the first two terms of the expansion, expanded $\left(1-\frac{1}{x}\right)^2$ and attempted a multiplication of terms. However, a significant number did not appreciate that the answer would come from adding three products and not just one or two. Some candidates appeared to misunderstand the question mistaking 'independent of *x*' for 'coefficient of *x*'.

Answers: **(i)** 2 **(ii)** –80

Question 7

(i) Some candidates did not realise that differentiation would be required to find a greatest distance. Candidates who did attempt differentiation were often handicapped by a lack of knowledge of the quotient rule - both the need for its use for this type of function and the actual formula and its application. Candidates who used the product rule rather than the quotient rule were often just as successful in obtaining a correct derivative but less so in obtaining a form that could be equated to zero and solved.

(ii) Most candidates realised that an expression for acceleration came from differentiating an expression for velocity, but combining the quotient (or product) rule and chain rule proved to be too complex for the majority of candidates.

Answers: **(i)** 0.5 **(ii)** –0.5

Question 8

(i) This question was very well answered by candidates using either the factor theorem or comparison of coefficients to find *p*. A minority of candidates using algebraic long division were less successful in finding a value for *p* (which could be done by equating $p + 22$ to *c*). Any errors that occurred in this question tended to come from slips with signs (often with the value of *c*) that could prove costly when candidates used comparison of coefficients throughout.

(ii) Most candidates obtained correct factors, but those who made further use of the factor theorem to find a solution of $\frac{1}{3}$ had difficulty using it to obtain the factor $(3x – 1)$. Candidates should be aware of the difference between being asked to solve an equation and being asked to find linear factors. Many needlessly went on to find solutions in this instance.

Answers: **(i)** $p = –26$, $a = 3$, $b = 11$, $c = –4$ **(ii)** $(x – 2)(3x – 1)(x + 4)$

Question 9

(i) There were many good solutions in both parts of this question. In this part most candidates appreciated that they had to add two arc lengths and two lengths equal to *AD* and nearly all candidates correctly obtained an arc length. Candidates who knew to use the cosine rule to obtain *AD* were often handicapped by mistakes and difficulties in evaluating the angle *AOD* correctly where they were expected to subtract $\frac{\pi}{6}$ from π, the angle on a straight line. Candidates need to be aware that when angles are given in radians it is usually easier and more accurate to work in radians than to attempt conversions to degrees. Candidates also need to be aware that care is needed when evaluating a cosine rule expression in order to avoid sign and order of operation errors. Care must also be taken to use the appropriate calculator mode. Candidates who prematurely rounded lengths lost accuracy in their final answer to this part.

(ii) Nearly all candidates found an area of a sector correctly. Candidates who had found the correct angle in part **(i)** then found it straightforward to evaluate the area of the triangle and many completely correct solutions were seen.

Answers: **(i)** 73.9 **(ii)** 231

Question 10

(i) Most candidates made a good start by using a correct trigonometric identity to obtain and solve an equation in either $\sec x$ or $\cos x$. A significant number of candidates did not recognise that $\sec x = 0$ had no solution, but most identified at least one solution for sec x = 2.

(ii) Most candidates started well and used an identity correctly to obtain an equation in $\cos^2 3y$, $\sin^2 3y$ or $\tan^2 3y$, but some did not understand that this was a squared trigonometric function and not a function of a function and were unable to make the step of square rooting. Candidates who did square root usually appreciated the order of operations and went on to divide an angle by three and obtain at least one solution. Candidates should be aware that there were four solutions to this equation arising from the positive and negative square roots of the appropriate trigonometric function. Candidates should be aware that if an answer is required in radians it is easier and usually more accurate to work with a calculator set in radian mode rather than attempt conversions between degrees and radians. Candidates should also be aware that answers are required to three significant figures.

(iii) Some candidates answered this question well. However, several otherwise good attempts were marred by rounding errors particularly those incurred when handling 1.9445. Although many candidates obtained a first solution they need to be aware that there could be a further solution within the given range, generated from $\pi - 0.4115$ and $2\pi + 0.4115$ and then subtracting $\frac{\pi}{4}$. Candidates need to understand the order of operations involved in this question. This did not seem to be fully understood. Candidates should be aware that in this question they will be less likely to make the error of mixing degrees and radians in the same expression if they work throughout in radians, with a calculator set in radian mode, rather than attempting conversions between degrees and radians.

Answers: **(i)** 60°, 300° **(ii)** 0.140, 0.907, 1.19, 1.95 **(iii)** 1.94, 5.91

Question 11

This is the last session where there will be an option in the final question.

Either

This was the less popular of the optional questions but candidates who attempted it often did well.

(i) Most serious attempts at this part were successful. One source of error was obtaining an incorrect final answer because of premature rounding in the straight line equation.

(ii) This proved to be a challenging part of this question, but most candidates realised that integration and use of limits needed to be employed. Candidates who had done those successfully often encountered difficulties with the manipulation required to obtain the final expression, needing to realise that e^{-a} could also be written as $\frac{1}{e^{a}}$ and that $e^{a} \times e^{a} = e^{2a}$.

(iii) Candidates who realised that they could attempt this part without having succeeded in previous parts could do well if they recognised the expression as a quadratic equation. Candidates need to be aware that not all equations written in terms of powers lend themselves to a solution involving the immediate taking of logs and that, on this occasion, prior factorisation and solution of a quadratic were necessary.

Answers: **(i)** 3.49 **(iii)** ln3

Or

(i) Candidates need to be aware that in questions where an answer is given they need to show fully the steps taken to arrive at the answer and that they should ensure that those steps are fully accurate. Most candidates made a good attempt at this part but candidates should be aware that no differentiation formulas are given in the formula list in the paper and that this topic requires thorough revision before an examination. Mistakes in the application of the quotient rule were made and also in the differentiation of exponential functions. Many candidates obtained a fully correct derivative but not all managed to simplify it to obtain the two correct products each of $6e^{4x}$ in the numerator that would clearly cancel each other out or an equivalent clearly correct method.

(ii) This part was well done with a very few candidates working with a normal instead of a tangent and a very few candidates producing an equation using a gradient still in terms of *x* rather than a numerical value. Candidates need to be aware that e^{0} is equal to 1.

(iii) There were many good responses, but not all candidates appreciated that reverse differentiation was required. There was some confusion in the use of integration signs and some candidates, having successfully integrated, used limits in the wrong expression. Candidates need to be aware that their use of limits should be clearly shown and that a substitution of a limit of zero does not always lead to an answer of zero.

Answers: **(i)** $A = 6$ **(ii)** $y - \frac{3}{2} = \frac{3x}{2}$ **(iii)** 0.2

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key point

Candidates are reminded of the importance of always including the constant of integration when performing an indefinite integral. Two questions on this paper relied heavily on its insertion and too many candidates lost valuable marks by their exclusion of the constant.

General comments

There were many impressive scripts indicating a good understanding of all subject areas. The marks were distributed over the whole mark range.

Candidates answered the following questions particularly well: **Question 2** (chain rule), **Question 3** (inequality), **Question 4** (binomial expansion) and **Question 11** (trig equations). The questions that proved most demanding were: **Question 5(ii)** (permutations and combinations), **Question 9** (relative velocity) and both parts of **Question 12** (calculus)

Comments on specific questions

Question 1

Few candidates were able to deal with the modulus concept successfully. They were expected to set up two linear equations, without the modulus sign, and solve them. Most candidates produced the straightforward equation $7x + 5 = 3x - 13$ and usually solved to get $x = -4.5$, although quite a number decided that the modulus meant that the answer had to be positive and so, incorrectly, gave a final answer of 4.5. Others attempted to set up four different equations and, inevitably, mistakes with the signs appeared. Too many candidates worked with the modulus sign throughout their work, making it difficult to decide whether they understood what they were meant to do. Many candidates chose to square both sides to set up and solve a quadratic equation. Errors in squaring and simplification meant that very few candidates were successful using this method.

Answer: $x = -4.5$ and $x = 0.8$.

Question 2

Overall this question was well attempted by a large number of candidates. Even weaker candidates often gained credit for differentiating the given formula. The weakest candidates, however, did not attempt to differentiate and so evaluated *A* when *r* = 6 and then tried to combine their answer with the given rate. A common error was to omit brackets when multiplying by $\frac{\mathrm{d}r}{\mathrm{d}t}$. Some candidates applied the chain rule with a term inverted incorrectly or used *r* = 5 A few thought that *r* was increasing at a rate of $6 - \frac{0.2}{\pi}$.

Answer: 6.8.

Question 3

Most candidates expanded the given expression and rearranged it into a quadratic. Where multiplication or division by –1 was used to obtain a positive coefficient for the x^2 term, the need to change the direction of the inequality symbol was sometimes ignored. Candidates usually factorised successfully, or used the formula, and for the majority of candidates the correct two critical values for *x* were obtained. Having obtained values, some candidates made no attempt to address the inequality aspect of the question and concluded with their two values of *x*. Many candidates either sketched a graph or constructed a table showing values either side of the critical values to determine the final solution set for *x*.

Answer: $0.5 < x < 3.5$

Question 4

(i) On the whole, this was very well done with most candidates including the binomial coefficient and the relevant power of 2. A few candidates made errors with the signs in the expansion and quite a few achieved –448 in the expansion but gave an answer of +448, apparently not realising that the minus sign was an important part of the coefficient.

(ii) Again well done with most candidates following through correctly from part **(i)**. A small number had problems with the signs.

Answers: **(i)** –448 **(ii)** 1792.

Question 5

(i) Many candidates correctly quoted and evaluated ${}^6P_4$. The most frequent incorrect alternative was ${}^6C_4$ and 6! was also often seen. A few candidates tried an exhaustive list but soon gave up the attempt.

(ii) This proved very challenging for many and there were few fully correct answers. There were a wide variety of approaches and again some tried exhaustive lists. Some candidates realised that there were two possible start numbers but then thought there were still 4 choices for the last digit. Those with some idea sometimes gave 5 x 4 as the inside digits whilst others used factorials for both inside and outside pairs.

Answers: **(i)** 360 **(ii)** 72

Question 6

(i) The vast majority of candidates recognised the need to express both the 8 and the 16 as powers of 2 and then combine the powers to obtain a linear equation. Many candidates manipulated their equation carefully and accurately to obtain the given linear equation. A common error involved a wrong sign, with the equation becoming $x - 3 - 6y - 9 = 4x - 4y$. Candidates whose first step was to write the original equation as $2^{x-3} = 16^{x-y} \times 8^{2y-3}$ avoided such sign problems. There were candidates who tried to combine the powers by erroneously writing $\frac{x-3}{6y-9} = 4x - 4y$ and then trying to manipulate this in the forlorn hope that the required equation would suddenly materialise.

(ii) Although most candidates expressed the 25 and the 125 as powers of 5, there was now no given equation to aim for and hence any errors in the work were not recoverable. Many candidates seemed to believe that they could then find the value of *x* and of *y* by considering only the equation obtained in this part and proceeded to write their equation in two different ways and attempt to solve simultaneously. A common incorrect method attempted by many candidates involved writing $\frac{5y}{5^{3x-6}} = \frac{5^2}{5^0}$ leading to $y = 2$ and $x = 2$.

Answer: **(ii)** $x = \frac{14}{9}$ and $y = \frac{2}{3}$

Question 7

(i) This was poorly attempted with a variety of responses often including combinations of tan, sec, x or 4x with some parts often being squared. The coefficient of 4 was often missing.

(ii) Very few seemed to realise that there was a link between parts **(i)** and **(ii).** The 4 frequently disappeared completely and often the 1 was not integrated.

(iii) Most candidates understood that they had to substitute the limits in and, mainly, did so correctly. Of those that got part **(ii)** correct very few were able to go on and manipulate the terms accurately. Too many chose to change terms in π to a decimal and found the arithmetic to be too difficult.

Answers: **(i)** $4\sec^2 4x$ **(ii)** $x + \frac{\tan 4x}{4}$ **(iii)** $\frac{1}{8}$

Question 8

(i) Most candidates correctly stated that the gradient was $\frac{7-4}{8-2}$ but many reduced this to $\frac{1}{3}$ rather than $\frac{1}{2}$. Use of $y = \frac{1}{2}x + c$ followed and this often resulted in an incorrect evaluation of the constant as 4 rather than 3. Many who got as far as $\lg y = \frac{1}{2}\lg x + 3$ could not see what the next step was and stopped, whilst some who knew to raise *x* to the half could not decide how to deal with the 3. A significant number did not get this far as they had used their point as (lg2, lg4) when trying to evaluate the constant.

(ii) and (iii) Very few candidates were able to proceed with these two parts and many candidates repeated their answers for the gradient and *y* intercept that they had found in part **(i).**

Answers: **(i)** $y = 1000x^{\frac{1}{2}}$ **(ii)** 1 **(iii)** 6

Question 9

Candidates found this question difficult. Completely correct solutions were rare and most candidates seemed to need more experience of this part of the syllabus. Many seemed not to understand the word 'bearing' and the vast majority could not translate the given information into a correct and helpful diagram.

Method marks were sometimes earned for attempts at using the sine rule and the cosine rule but, in many other cases, no marks could be awarded as the triangles being used were right-angled or included reflex angles. Often a mixture of speed and distance were used in a candidate's workings in trying to find an angle.

Answers: **(i)** 223° **(ii)** 2 hours 5 minutes (or 2.09 hrs)

Question 10

(i) Most of the average and better candidates realised the need to integrate the acceleration to get the velocity. However, a significant number used equations of motion with constant acceleration. Most integrated the two terms correctly but many did not introduce the constant of integration. These candidates either solved one of the quadratic equations $4t - t^2 = 0$ or $4t - t^2 = 12$. Another error in the solution of the second quadratic often fortuitously resulted in the 'correct' value of *t*.

(ii) Those who got the correct solution to part **(i)** almost always went on to get the correct answer to part **(ii)** although poor arithmetic let down a few. Those who forgot the constant in part **(i)** usually integrated the two terms successfully.

Answers: **(i)** 6 **(ii)** 72m

Question 11

(a) There were many good solutions to this part and most attempted to obtain tan from sin and cos. However, a number got the expression inverted and many also lost the minus sign. Most of those achieving $\tan x = -\frac{9}{4}$ went on to get the correct answers, although some used 66° as one of their answers and gave angles in the wrong quadrants.

(b) This proved the most accessible part of the question. Cosec*y* became $\frac{1}{\sin y}$ correctly most of the time although the subsequent rearrangement to a quadratic sometimes lost a term. Some confused $\frac{1}{\sin y} - 1$ replacing it with $\frac{1}{\sin y - 1}$. Factorisation was usually correct and the answers were achieved without much difficulty. There were a number of cases of incorrect rounding and insufficient accuracy for the answers.

(c) Rearranging the given equation to $\cos\left(\frac{z}{3}\right) = \frac{3}{5}$ proved challenging for many with cos without an angle being seen, and 3 being taken from within the function to the other side of the equation. Some seemed to think that $\cos^{-1}$ is the same as $\frac{1}{\cos}$. A significant number gave the initial solution(s) in degrees but many of these then converted to radians, thus gaining the mark belatedly. Quite a few divided (their) 0.927 by 3 instead of multiplying. Extra solutions, both in and out of the range, were quite common.

Answers: **(a)** 114° and 294° **(b)** 14.5°, 165.5°, 199.5° and 340.5° **(c)** 2.78 and 16.1.

Question 12

This is the last session where there will be an option in the final question.

Either

(i) Of those who realised that they needed to find the equation of the curve by integration, many made errors with the coefficient of $e^{-\frac{x}{4}}$ and usually also omitted to include, and hence find a value for, the constant of integration. A common slip involved the substitution of $x = -4$ in $e^{-\frac{x}{4}}$ giving e^{-1}. Weaker candidates did not realise the need to integrate and tried to use the given expression for the gradient as the gradient of a line.

(ii) Many did not appreciate that the gradient of each tangent was available immediately from the initial statement involving the derivative. There were many instances of equations being formed that involved exponential functions rather than linear equations. Other candidates seemed to think that the gradient of the line *AB* was an appropriate starting point or thought that the midpoint of *AB* was needed. A number did not leave their coordinate in terms of e.

Answers: **(i)** $14 - 4e$ **(ii)** $\frac{4}{1-e}$

Or

Candidates struggled with the use of minus signs in solutions to both parts.

(i) Despite the evidence of the diagram, some candidates were happy to give a positive x-coordinate for Q. This often occurred after a sign slip in the differentiation or from an inability to evaluate the gradient of the normal correctly.

(ii) Problems with signs occurred either as the result of incorrect integration or from getting in a muddle with the application of the limits. Candidates answering this question seemed less willing to retain e throughout solutions and decimal approximations were common. As a result, some candidates with otherwise sound work throughout did not obtain the exact value for the area.

Answers: **(i)** (–3 , 0) **(ii)** 3

ADDITIONAL MATHEMATICS

Paper 4037/23
Paper 23

Key messages

Candidates need to read the questions carefully before starting their solutions. This should avoid time and effort being wasted on an attempt involving an inappropriate method for which no marks can be awarded. It should also avoid answers being given in the wrong format. They should also remember that, in some questions, sketches can provide valuable assistance.

General comments

This paper produced a full range of marks. Candidates were usually very successful when answering **Questions 1 (modulus equation)**, **2 (matrices)**, **3 (simplification of surds)** and **11 (logarithms)**, whilst **Questions 4 (vectors)**, **7 (integration)**, **9 (transformation to straight line graph)** and **10(ii) (stationary value)** were often less rewarding. Several candidates penalised themselves with poor presentation. This was most evident in **Question 11(iii)** where "2" and "*z*" were miscopied, usually for each other but sometimes as a "7".

There was no evidence of any problems with the time allowed to complete the paper. Very few candidates had difficulty completing their answers in the space provided. There was some deleted work. Erasing and overwriting work should be avoided since it can make it difficult to determine the candidate's intentions.

Comments on specific questions

Question 1

The most popular method was to solve the equations $5x + 7 = 13$ and $5x + 7 = -13$. Several candidates only solved the first of these equations, whilst a much smaller proportion only solved the second. Most errors stemmed from mistakes involving signs, e.g. $-5x + 7 = 13$. A few candidates gave their final answer as $x = \pm\left|\frac{6}{5}\right|$ or $|x| = 1.2$. Others calculated -4 correctly but then gave $+4$ as their answer. A small number of candidates squared both sides of the given equation and factorised the resulting quadratic. This was a longer, but generally more successful, method.

Answers: $x = 1.2$ or -4.

Question 2

This proved to be one of the more popular questions involving matrices.

(i) Usually answered correctly.

(ii) Most candidates understood how to express the simultaneous equations in matrix form and pre-multiplied $\begin{pmatrix} 39 \\ 23 \end{pmatrix}$ by their inverse. Only a very small number of candidates tried to post-multiply. An appreciable number of candidates ignored the instruction to use their answer to part **(i)**.

Answers: **(i)** $\frac{1}{10}\begin{pmatrix} 6 & -8 \\ -4 & 7 \end{pmatrix}$ **(ii)** $x = 5$, $y = 0.5$.

Question 3

The majority of candidates understood the methods required for this question and produced good responses. Squaring $3\sqrt{3}$ sometimes resulted in either 9 or 81 and, less frequently, the $-6\sqrt{3}$ was omitted. Several candidates made the multiplication more involved by not collecting the integer terms, thus multiplying $(27 - 6\sqrt{3} + 1)$ by $(2\sqrt{3} + 3)$.

Answer: $\frac{38\sqrt{3}+48}{3}$.

Question 4

Many candidates find questions involving vectors quite challenging. The most popular approaches for calculating $\overrightarrow{OY}$ were to either find $\overrightarrow{XZ}$ and add $\frac{1}{4}$ of this to $\overrightarrow{OX}$ or substitute $\left(\overrightarrow{XO}+\overrightarrow{OY}\right)$ and $\left(\overrightarrow{YO}+\overrightarrow{OZ}\right)$ for $\overrightarrow{XY}$ and $\overrightarrow{YZ}$ respectively in the given equation. Sometimes $\frac{1}{3}$ was used instead of $\frac{1}{4}$ in the former. Most mistakes followed either sign errors or the careless use of brackets. Only a few candidates used the ratio theorem. Several candidates, having calculated $\overrightarrow{OY}$, did not continue to determine the unit vector in the direction of $\overrightarrow{OY}$.

Answer: $\begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix}$.

Question 5

Answers to this question varied considerably. Many candidates appreciated the steps needed and quickly arrived at the correct answer whilst others made little or no progress. Most mistakes stemmed from the lack of a clear understanding of the coefficients in a quadratic equation. Sometimes an attempt was made to combine the two terms involving m, in other answers 1 was used instead of m for the coefficient of x^2. Several answers involved 1 and 49 in either two orderings or one incorrect ordering. A small number did not give an ordering.

Answers: $1 < m < 49$.

Question 6

(a) Attempts involving the drawing of a right-angled triangle were less frequent than those involving the application of a trigonometric expression. $\tan^2 x = \frac{1-\cos^2 x}{\cos^2 x}$ and $\tan^2 x = \frac{1}{\cos^2 x} - 1$ appeared equally popular. Several candidates did not realise that all of the trigonometric ratios had to be written in terms of p, so some answers involved a mixture of the two, e.g. $\frac{1}{p\cos x} - 1$.

(b) This trigonometric identity resulted in more correct proofs than usual. Nearly all candidates worked from the left-hand side to the right-hand side. Those candidates who worked with $\tan\theta$ and $\cot\theta$ generally made better progress than those working with $\sin\theta$ and $\cos\theta$. Quite often the latter reached $\frac{1}{\sin^2 x\cos^2 x}$ and could make no further progress.

Answers: **(a)** $\frac{1-p^2}{p^2}$

Question 7

A number of candidates found both parts of this question challenging. Whilst there were many correct answers to either one or both parts, a considerable number of candidates did not realise that the integrand had to be written in index form before attempting to integrate.

(a) A rather high proportion of candidates assumed that $\int f(x)g(x)dx = \int f(x)dx \times \int g(x)dx$, resulting in little valid progress. Those multiplying out the bracket before integrating usually recognised that $\sqrt{x} = x^{\frac{1}{2}}$, but sometimes had difficulty multiplying this by *x*. Occasionally arithmetic mistakes were made when dividing by a fraction.

(b) In this part several candidates assumed that $\int \frac{f(x)}{g(x)}dx = \frac{\int f(x)dx}{\int g(x)dx}$, often leading to a term involving '*x*' in the numerator. The second stage, substituting limits in the correct order and subtracting, was usually applied correctly to their integral.

Answers: **(a)** $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$ **(b)** 1.6.

Question 8

The majority of candidates were able to make good progress with this question, usually gaining most, if not all, of the marks available. Weaker candidates had difficulty understanding the question, some not realising that the vertices of the trapezium must be read in order. In many cases a small sketch might have helped to identify which gradients and lines were appropriate. Nearly all candidates were aware that the line perpendicular to a line with gradient *m* has gradient $\frac{-1}{m}$. Most candidates found the point of intersection of the lines *AD* and *CD*, although several, often without a correct sketch, used an incorrect pair. Several attempts involved the incorrect assumption that the mid-points of *AC* and *BD* are the same. Having found the coordinates of *D*, most candidates were able to calculate the required area, the 'array' method being preferred to the formula for the area of a trapezium. A few candidates did not repeat the first point in their array and this often led to the use of six, not eight, products.

Answer: 20.

Question 9

A rather high proportion of candidates did not understand that, in order to obtain x^2y on the left-hand side of the equation, it was necessary to multiply the equation by x^2. This resulted in most of the remaining marks in the question becoming unattainable. There were several sign errors in parts **(ii)** and **(iii)**.

(i) Whilst most correct answers stated the variable to be x^3, some consisted of bx^3 or $x^2y = a + bx^3$. These were condoned.

(ii) This was usually well answered by those candidates who had the correct variable. The most popular scales were either 5 or 10 units to 2 cm on the x^2y-axis and either 8 or 10 units to 2 cm on the x^3-axis. Some scales did not allow for all of the values to be plotted.

(iii) Those candidates with correct answers to parts **(i)** and **(ii)** usually gave the correct value for *a* and calculated the correct value for *b*. Occasionally the values were interchanged.

(iv) Those with a good understanding of this topic usually calculated a correct value for *y* when *x* = 3.7. Substituting 3.7 for *x*, together with their values for *a* and *b*, was more popular than reading the value of x^2y corresponding to 3.7^3 and dividing this by 3.7^2. Quite a few attempts at the latter involved 56.53 instead of 50.653. Weaker candidates read off the value on the vertical axis which corresponded to the value 3.7 on the horizontal axis. Several wrong answers included no working, so no credit could be given.

Answers: **(i)** x^3 **(iii)** a = 10, b = −0.6 **(iv)** −1.48.

Question 10

(i) Nearly every candidate attempting this part showed that the square root followed from the application of Pythagoras to find the distance *CD*. Most candidates were able to indicate that the reason for the fractions stemmed from time = distance ÷ speed.

(ii) Responses to this part of the question varied considerably. Strong candidates obtained the correct answers, whilst the weaker ones tried to solve $T = 0$. Several candidates made the differentiation more involved by treating the two terms as quotients, or combining the two terms into a single quotient. Several attempts to differentiate omitted the '$2x$' from $\frac{\mathrm{d}(x^2+6400)}{\mathrm{d}x}$.

Answers: **(ii)** $x = 60$, $T = 30\frac{2}{3}$.

Question 11

Most candidates were able to make a good attempt to answer at least one part of this question.

(a) This part was usually answered well. A common error was to use $\sqrt{100}$ instead of 100^2. There were some arithmetic errors following e.g. $\frac{1}{2}x - 1 = 6.644$, when the order of adding one and multiplying by two was reversed or one of the operations was omitted or the inverse operation applied. In some answers the working was not accurate enough, resulting in the answer 15.2.

(b) Again, candidates usually answered this part well. The most frequent mistake was to assume that $\log_y 2 + \log_y 256 = \log_y 258$. Another error seen on more than one occasion was to obtain $y^3 = 512$ and then take the square root instead of the cube root.

(c) Nearly all attempts correctly expressed the terms in the given equation as powers of 6, only a few trying to cancel e.g. 216 and 36. Most candidates continued to either add or subtract the indices as appropriate and solve their equation to obtain the correct value for *z*. A common mistake occurred with the sign when subtracting $(6 - 2z)$. Weaker solutions reached a correct equation and then divided the indices instead of subtracting them.

Answers: **(a)** 15.3 **(b)** 8 **(c)** 3.5.

Question 12

This is the last session where there will be an option in the final question. On this occasion, the Either option was more popular than Or option.

Either

Responses for part **(iii)** were usually better than those for the other two parts.

(i) Several candidates attempting this part of the question gave their answer in the format $a(x + b)^2 + c$, rather than that requested in the question. Working from the given quadratic expression to the format required by completing the square was more popular than expanding $(ax + b)^2 + c$ and comparing coefficients. The algebraic manipulation was rather demanding with mistakes such as $(ax)^2 = ax^2$ featuring in several responses.

(ii) Nearly all of those candidates who recognised the connection between this and the first part of the question had little difficulty finding the inverse of the function f. Most of the candidates starting afresh produced an expression in which *x* appeared more than once. A few candidates who started again were successful, occasionally obtaining the correct answer here after an incorrect answer in part **(i)**.

(iii) The majority of answers involved substituting $\frac{1}{x}$ for x in the function f rather than its equivalent expression derived in part **(i)**. Only a small proportion of candidates rejected the negative solution to their quadratic equation.

Answers: **(i)** $(2x + 8)^2 - 9$ **(ii)** $f^{-1}(x) = \frac{\sqrt{x+9}-8}{2}$ **(iii)** $x = 0.5$.

Or

(i) Several candidates did not attempt this part of the question. However, there were many correct answers. A frequent incorrect answer was the integer 4.

(ii) This was very well answered with only a few candidates either making a sign error or confusing squaring with taking a square root. Occasionally an answer was left in terms of y rather than x.

(iii) Nearly all answers included a correct quadratic equation. This was usually solved correctly but quite often the root less than 2 was not rejected.

(iv) The majority of candidates substituted correctly and obtained the expression $\frac{3\left(\frac{3x-4}{x-2}\right)-4}{\left(\frac{3x-4}{x-2}\right)-2}$. Most of these made a good attempt to transform this into the format required.

Answers: **(i)** 3.5 **(ii)** $\frac{x^2+7}{2}$ **(iii)** $x = 4$ **(iv)** $k^2: x \rightarrow 5 - \frac{4}{x}$.